Tilt Sensing Using Linear Accelerometers

by: **Kimberly Tuck**
Accelerometer Systems and Applications Engineering
Tempe, AZ

INTRODUCTION

This application note explains the importance of understanding how to acquire a reliable and accurate tilt reading for accelerometer applications by comparing the advantages and disadvantages of various tilt measurement techniques. Accelerometers used for tilt sensing require high resolution to meet the demands of many new emerging applications such as tilt enabled computer mouse/pointers, motion enabled video game solutions and PDA-cell phone/mp3 player screen navigation.

The overall benefit of the accelerometer for tilting applications used in PDAs for screen navigations is a new method to view, scroll, select and move with a minimum number of buttons required. This concept affords a PDA with a larger screen area for viewing. Navigation through menus is made easier with the ability to make selections based on tilt. The choices are highlighted and then can be selected either by using a physical “execute” button on the PDA or by using click or double click tap detection of the accelerometer. The user can make selections in a menu driven environment this way. Also the accelerometer can also be used to sense the tilt of the PDA to change from landscape to portrait using gravity to change the screen orientation for viewing.

Interactive video games are becoming increasingly popular. Accelerometers are used to detect the tilting motions of the joystick for the game.This has created games where the user can feel more immersed in the game.

Tilt is a static measurement. The force of gravity is used as an input to determine the orientation of an object calculating the degree of tilt.The accelerometer will experience acceleration in the range from -1g to +1g through 180° of tilt.

$$1g = -9.8\ m/s^2$$

0G OFFSET CALIBRATION

Accuracy and repeatability is a general concern for nearly all accelerometer applications. The accuracy of the tilt measurement can be improved by using a 0g-offset calibration technique to compensate for offset errors. Refer to Freescale application note AN3447, “Implementing Auto-Zero Calibration Technique for Accelerometers.” Even though the offset is trimmed, offset can shift due to packaging stresses, aging and external mechanical stresses due to mounting and orientation.This results in offset calibration error. It is important to implement a 0g calibration routine for the accelerometer to compensate for the 0g offset.

MEASUREMENT TECHNIQUES

This section discusses the different ways to implement tilt comparing different ways to measure the corresponding angle from the acceleration output.

Measuring Tilt using One Axis

In the case of a dual-axis accelerometer (XY) mounted perpendicular to gravity the tilt algorithm is limited to one axis of sensitivity. As shown in Figure 1 the accelerometer is tilted along the X-axis. The Y-axis remains at 0g output throughout the full rotation of the X-axis in this case.

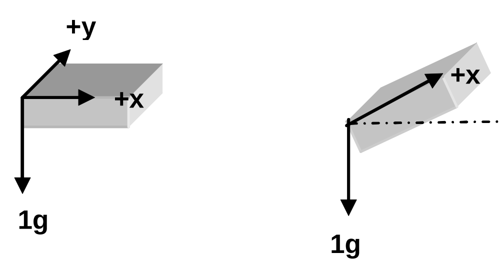

Figure 1. Dual-Axis Accelerometer with One Axis of Tilt

If one axis (X-axis) is used to calculate the tilted angle of the accelerometer the following trigonometry relationship is used:

$$V_{OUTX} = V_{OFF} + S \times \sin\theta$$

Where: V_{OUTx} is the voltage output from the X-axis of the accelerometer, V_{OFF} is the offset voltage, and S is the sensitivity of the accelerometer.

The acceleration output on the X-axis due to gravity is equal to the following:

$$A_X = \frac{V_{OUTX} - V_{OFF}}{S}$$

In order to solve for the angle of tilt the equation becomes the following:

$$\theta = \sin^{-1}(A_X)$$

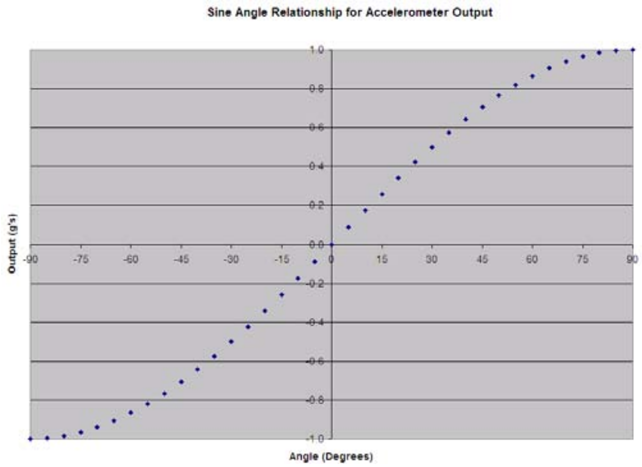

Figure 2. Accelerometer Output (g's) Tilting from -90° to +90° with a One Axis Measurement

This graph shows the output in g's of the accelerometer as it tilts from -90° to +90°. Notice that the tilt sensitivity diminishes between -90° and -45° and between +45° to +90°. This resolution problem between these values makes this method of calculating the angle of tilt inaccurate when the accelerometer output is near the +1g or -1g range. A dual-axis accelerometer horizontally mounted would be limited by this method of calculating tilt and would not be accurate over a 360° rotation. It would only be useful for angle measurements between -45° to +45° of tilt.

Another disadvantage of the single axis measurement tilt technique is that it is impossible to know the difference between two tilt angles that result in the same sensor output. The output is a sine function, so for example it would be impossible to know from a 0.5g output reading if the accelerometer was tilted 30° or 150° by looking at the accelerometer output. One would have to be aware of the correct orientation of the accelerometer and have a sense for the quadrant of tilt. This disadvantage is overcome by using a two axis measurement tilt technique and is explained in the next section.

Measuring Tilt using a Two Axis Solution

The resolution problems and tilt orientation difficulties can be addressed by mounting the accelerometer vertically so that the Y-axis is parallel to gravity, or by using a tri-axis accelerometer using at least 2 of the 3 axis. Using more than one axis to calculate tilt produces a more accurate solution.

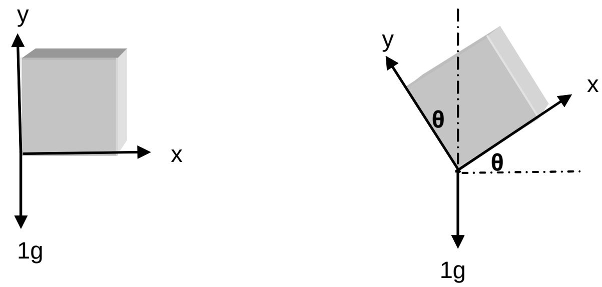

Figure 3. Using a (Dual- or Tri-Axis) Accelerometer with Two Axes for Measuring Tilt

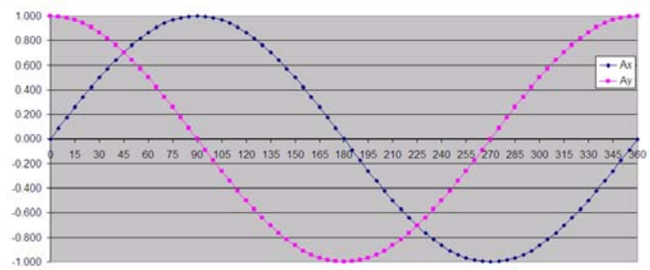

Figure 4. Sine Function of the X Output and Cosine Function of the Y Output

The graph above shows that when using a two axis solution the component due to gravity on the X-axis follows the sine function while the component due to gravity acting on the Y-axis follows the cosine function. Notice that the tilt sensitivity (slope of the line) in the X-direction is at its maximum while the Y-sensitivity is at its minimum and visa versa. Therefore the maximum tilt sensitivity can be maintained if both the X and the Y outputs are combined.

Table 1 displays 360° of tilt with the acceleration output of the X component and Y components due to gravity. Also the change in gravity with the change in angle is analyzed through the full rotation for both components. The two sensitivities are combined which results in a constant output of 17.45mg/°.

Table 1. Tilt using the X and Y-axis

Angle (°)	A_X (g's)	dg/dDeg A_X TS_X(mg/°)	A_Y (g's)	dg/dDeg A_Y TS_Y(mg/°)	sqrt(TS_X^2+TS_Y^2) (mg)	sqrt(A_X^2+A_Y^2) (g)
0	0.000	17.452	1.000	-0.152	17.45	1.00
30	0.500	15.038	0.866	-8.858	17.45	1.00
60	0.866	8.594	0.500	-15.190	17.45	1.00
90	1.000	-0.152	0.000	-17.452	17.45	1.00
120	0.866	-8.858	-0.500	-15.038	17.45	1.00
150	0.500	-15.190	-0.866	-8.594	17.45	1.00
180	0.000	-17.452	-1.000	0.152	17.45	1.00
210	-0.500	-15.038	-0.866	8.858	17.45	1.00
240	-0.866	-8.594	-0.500	15.190	17.45	1.00
270	-1.000	0.152	0.000	17.452	17.45	1.00
300	-0.866	8.858	0.500	15.038	17.45	1.00
330	-0.500	15.190	0.866	8.594	17.45	1.00

Basic Trigonometry

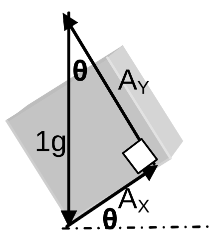

Figure 5. Basic Trigonometry

The acceleration in the X-axis in Table 1 is calculated by the following equation:

$$A_X = \sin\theta$$

The acceleration on the Y-axis is calculated with:

$$A_Y = \cos\theta$$

If the combination of the X acceleration and the Y acceleration is used:

$$\frac{A_X}{A_Y} = \tan\theta$$

The tilt sensitivity equation mg/° was calculated by taking the difference between the acceleration output between 1 degree at that point. For example, the tilt sensitivity at 15° is calculated by the following:

$$\sin(16) - \sin(15) = 16.818$$

The Y-axis is 90° from the X-axis and therefore it makes sense that the Y-axis experiences a 1g acceleration while the X-axis experiences a 0g acceleration. The combined acceleration is always 1g.

$$A = \sqrt{A_X^2 + A_Y^2} = 1g$$

The sensor is most responsive to changes in tilt when the sensitive axis is perpendicular to the force of gravity. When perpendicular to the force of gravity the accelerometer experiences approximately 17.45mg per degree tilt. It is least responsive when the sensitive axis is parallel to the force of gravity in the +1g or -1g orientation, with a responsiveness of only 0.15mg per degree of tilt. This is clearly displayed in Figure 6 where the absolute value of the tilt sensitivity was taken. As the X-axis is at its minimum tilt sensitivity the Y-axis is at its maximum tilt sensitivity. By combining the X and Y-axis solving for the tilt angle using arctan (A_X/A_Y), a constant tilt sensitivity of 17.45mg can be maintained through a 360° rotation.

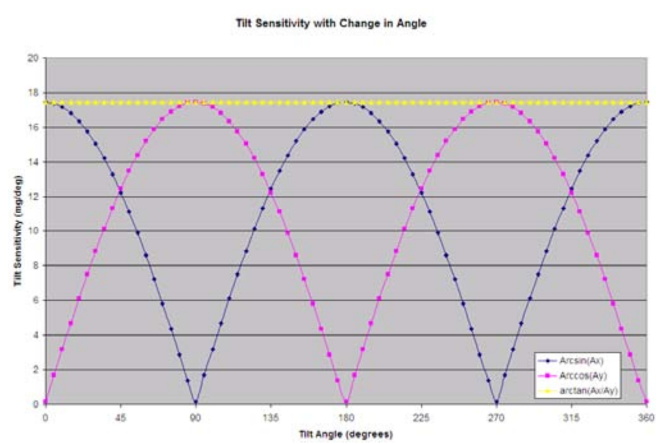

Figure 6. Tilt Sensitivity versus Tilt Angle

Quadrant Orientation

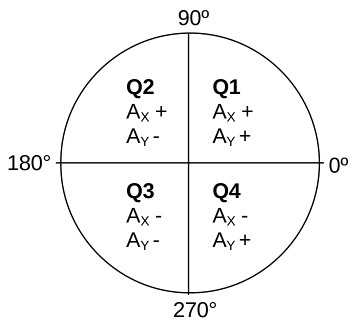

Figure 7. Quadrants of a 360 Degree Rotation

It is important to know the sign of the X and Y accelerations to determine the quadrant of tilt that is applicable because the outputs from the first and third quadrant will be the same and the outputs from the second and fourth quadrant will also be the same. For example tan (45) = 1 and tan (225) = 1. When taking the arctan of a positive value the tilt angle is in either the first or third quadrant. Knowing the sign of A_X and A_Y will determine exactly which quadrant. When taking the arctan of a negative value the tilt angle is in either the second or fourth quadrant. Knowing the sign of A_X and A_Y will determine exactly which quadrant the accelerometer is tilting through.

If in Quadrant 1 = arctan (A_X/A_Y)

If in Quadrant 2 = arctan (A_X/A_Y) + 180

If in Quadrant 3 = arctan (A_X/A_Y) + 180

If in Quadrant 4 = arctan (A_X/A_Y) + 360

Measuring Tilt using a Three Axis Solution

In order to define the angles of the accelerometer in three dimensions the pitch, roll and theta are sensed using all three outputs of the accelerometer. Pitch (ρ) is defined as the angle of the X-axis relative to ground. Roll (φ) is defined as the angle of the Y-axis relative to the ground. Theta (θ) is the angle of the Z axis relative to gravity.

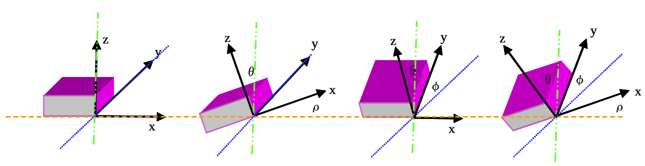

Figure 8. Three Axis for Measuring Tilt

$$\rho = \arctan\left(\frac{A_X}{\sqrt{A_Y^2 + A_Z^2}}\right)$$

$$\phi = \arctan\left(\frac{A_Y}{\sqrt{A_X^2 + A_Z^2}}\right)$$

$$\theta = \arctan\left(\frac{\sqrt{A_X^2 + A_Y^2}}{A_Z}\right)$$

Now the acceleration due to gravity on the X-axis, Y-axis and Z-axis are combined. The resultant sum of the accelerations from the three axes is equal to 1g when the accelerometer is static.

$$\sqrt{A_X^2 + A_Y^2 + A_Z^2} = 1g$$

A/D Converter Resolution Limitations

Discrete values are used when the signal is digitized and therefore the resolution is limited by the number of bits in the A/D converter. Table 2 displays the 8-bit A/D converter values for the X and Z-axis assuming an ideal rotation about the y axis.

The 3.3V supply voltage is divided by 255 (2^8-1) steps from the A/D converter. This value is divided by the sensitivity of 0.8V/g to solve for the acceleration due to gravity at each step.

$$\frac{3.3V}{255 \times 0.8mV/g} = 16.176mg$$

Therefore each increasing bit will account for an additional 16.176mg.

From Table 2 it can be seen that a single axis solution will produce a decreasing resolution as the device is tilted from 0° to 90°, but a two axis solution will produce a fairly steady resolution throughout the entire tilt range.

The angle calculation based on acceleration of a single axis is the following:

$$\theta = \sin^{-1}(A_X)$$

The resolution goes from 0.927 degrees to 9.332, which is unacceptable for a tilt application.The resolution gets increasingly worse through the tilt.

The angle calculation based on acceleration of two axes is the following:

$$\theta = \tan^{-1}\left(\frac{A_X}{A_Z}\right)$$

The resolution is between 0.748° - 1.317° throughout the entire tilt range. Again this shows the improved accuracy of using two axes to calculate tilt. Figure 6 displays the comparison of these two methods using the 8-bit A/D converter.

NOTE: The same analysis applies for angles from 91° to 360° in the other three quadrants.

Using a 10-bit A/D converter the 3.3V supply voltage is divided by 1023 (2^{10}-1) steps from the A/D converter. This value is then divided by the sensitivity of 0.8V/g to solve for the acceleration due to gravity at each step.

$$\frac{3.3V}{1023 \times 0.8mV/g} = 4.032mg$$

Using a 10-bit A/D converter with a 2 axis solution the resolution is between 0.171 and 0.327 throughout the tilt range, while the 1 axis solution resolution starts out at 0.231 at 0° and increases to 5.147 as it approaches 90°. A higher resolution is achievable with a bigger A/D converter. The comparison using the 10-bit A/D converter is shown in Figure 10.

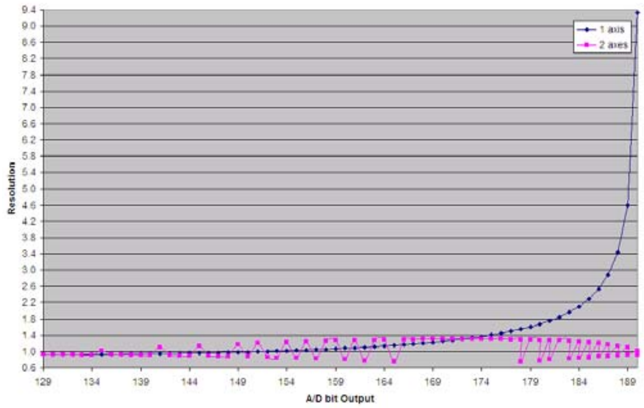

Figure 9. Tilt Resolution for a One or Two axis Tilt Algorithm Using an 8-Bit A/D Converter

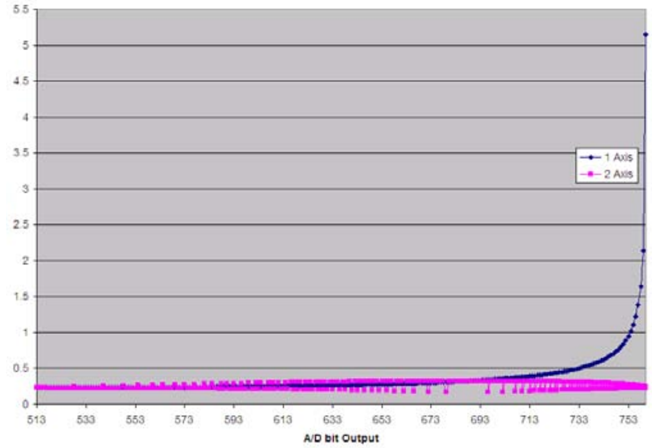

Figure 10. Tilt Resolution for a One or Two Axis Tilt Algorithm Using a 10-Bit A/D Converter

Table 2. A/D converter values for A_X and A_Z for tilt from 0° to 90°

A/D A_X	Ax- g's	A/D A_Z	Az-g's	Angle 1-Axis	Angle 2-Axes	Resolution 1-Axis	Resolution 2=Axis
128	0.0000	190	1.0029	0.0000	0.0000	0.9269	0.9240
129	0.0162	190	1.0029	0.9269	0.9240	0.9269	0.9240
130	0.0324	190	1.0029	1.8540	1.8476	0.9271	0.9236
131	0.0485	190	1.0029	2.7816	2.7702	0.9276	0.9226
132	0.0647	190	1.0029	3.7100	3.6914	0.9283	0.9212
133	0.0809	190	1.0029	4.6393	4.6106	0.9293	0.9193
134	0.0971	190	1.0029	5.5698	5.5275	0.9305	0.9169
135	0.1132	189	0.9868	6.5018	6.5463	0.9320	1.0188
136	0.1294	189	0.9868	7.4356	7.4716	0.9338	0.9253
137	0.1456	189	0.9868	8.3713	8.3929	0.9357	0.9214
138	0.1618	189	0.9868	9.3093	9.3099	0.9380	0.9170
139	0.1779	189	0.9868	10.2499	10.2222	0.9405	0.9122
140	0.1941	189	0.9868	11.1932	11.1292	0.9433	0.9070
141	0.2103	188	0.9706	12.1396	12.2251	0.9464	1.0959
142	0.2265	188	0.9706	13.0894	13.1340	0.9498	0.9089
143	0.2426	188	0.9706	14.0428	14.0362	0.9535	0.9022
144	0.2588	188	0.9706	15.0003	14.9314	0.9574	0.8952
145	0.2750	187	0.9544	15.9620	16.0736	0.9617	1.1422
146	0.2912	187	0.9544	16.9284	16.9661	0.9664	0.8926
147	0.3074	187	0.9544	17.8998	17.8503	0.9714	0.8842
148	0.3235	187	0.9544	18.8765	18.7258	0.9767	0.8755
149	0.3397	186	0.9382	19.8590	19.9037	0.9825	1.1780
150	0.3559	186	0.9382	20.8475	20.7723	0.9886	0.8685
151	0.3721	185	0.9221	21.8426	21.9745	0.9951	1.2023
152	0.3882	185	0.9221	22.8447	22.8337	1.0021	0.8591
153	0.4044	185	0.9221	23.8543	23.6821	1.0095	0.8484
154	0.4206	184	0.9059	24.8717	24.9048	1.0175	1.2227
155	0.4368	184	0.9059	25.8976	25.7407	1.0259	0.8359
156	0.4529	183	0.8897	26.9325	26.9802	1.0349	1.2395
157	0.4691	183	0.8897	27.9770	27.8015	1.0445	0.8212

Table 2. A/D converter values for A_X and A_Z for tilt from 0° to 90° (continued)

A/D A_X	Ax- g's	A/D A_Z	Az-g's	Angle 1-Axis	Angle 2-Axes	Resolution 1-Axis	Resolution 2=Axis
158	0.4853	182	0.8735	29.0317	29.0546	1.0547	1.2531
159	0.5015	181	0.8574	30.0973	30.3236	1.0656	1.2690
160	0.5176	181	0.8574	31.1746	31.1225	1.0772	0.7989
161	0.5338	180	0.8412	32.2642	32.3998	1.0896	1.2774
162	0.5500	180	0.8412	33.3670	33.1785	1.1029	0.7787
163	0.5662	179	0.8250	34.4840	34.4608	1.1170	1.2823
164	0.5824	178	0.8088	35.6162	35.7539	1.1322	1.2931
165	0.5985	178	0.8088	36.7646	36.5014	1.1484	0.7476
166	0.6147	177	0.7926	37.9306	37.7939	1.1659	1.2925
167	0.6309	176	0.7765	39.1153	39.0939	1.1847	1.2999
168	0.6471	175	0.7603	40.3202	40.3999	1.2050	1.3060
169	0.6632	174	0.7441	41.5471	41.7108	1.2269	1.3109
170	0.6794	173	0.7279	42.7977	43.0251	1.2506	1.3143
171	0.6956	172	0.7118	44.0741	44.3415	1.2764	1.3164
172	0.7118	171	0.6956	45.3787	45.6585	1.3046	1.3171
173	0.7279	170	0.6794	46.7141	46.9749	1.3354	1.3164
174	0.7441	169	0.6632	48.0834	48.2892	1.3693	1.3143
175	0.7603	168	0.6471	49.4901	49.6001	1.4068	1.3109
176	0.7765	167	0.6309	50.9386	50.9061	1.4484	1.3060
177	0.7926	166	0.6147	52.4336	52.2061	1.4950	1.2999
178	0.8088	165	0.5985	53.9811	53.4986	1.5476	1.2925
178	0.8088	164	0.5824	53.9811	54.2461	1.5476	0.7476
179	0.8250	163	0.5662	55.5885	55.5392	1.6073	1.2931
180	0.8412	162	0.5500	57.2646	56.8215	1.6761	1.2823
180	0.8412	161	0.5338	57.2646	57.6002	1.6761	0.7787
181	0.8574	160	0.5176	59.0207	58.8775	1.7561	1.2774
181	0.8574	159	0.5015	59.0207	59.6764	1.7561	0.7989
182	0.8735	158	0.4853	60.8714	60.9454	1.8507	1.2690
183	0.8897	157	0.4691	62.8363	62.1985	1.9649	1.2531
183	0.8897	156	0.4529	62.8363	63.0198	1.9649	0.8212
184	0.9059	155	0.4368	64.9424	64.2593	2.1061	1.2395
184	0.9059	154	0.4206	64.9424	65.0952	2.1061	0.8359
185	0.9221	153	0.4044	67.2289	66.3179	2.2866	1.2227
185	0.9221	152	0.3882	67.2289	67.1663	2.2866	0.8484
185	0.9221	151	0.3721	67.2289	68.0255	2.2866	0.8591
186	0.9382	150	0.3559	69.7573	69.2277	2.5283	1.2023
186	0.9382	149	0.3397	69.7573	70.0963	2.5283	0.8685
187	0.9544	148	0.3235	72.6329	71.2742	2.8756	1.1780
187	0.9544	147	0.3074	72.6329	72.1497	2.8756	0.8755
187	0.9544	146	0.2912	72.6329	73.0339	2.8756	0.8842
187	0.9544	145	0.2750	72.6329	73.9264	2.8756	0.8926
188	0.9706	144	0.2588	76.0694	75.0686	3.4366	1.1422
188	0.9706	143	0.2426	76.0694	75.9638	3.4366	0.8952
188	0.9706	142	0.2265	76.0694	76.8660	3.4366	0.9022
188	0.9706	141	0.2103	76.0694	77.7749	3.4366	0.9089
189	0.9868	140	0.1941	80.6678	78.8708	4.5983	1.0959

Table 2. A/D converter values for A_X and A_Z for tilt from 0° to 90° (continued)

A/D A_X	Ax- g's	A/D A_Z	Az-g's	Angle 1-Axis	Angle 2-Axes	Resolution 1-Axis	Resolution 2=Axis
189	0.9868	139	0.1779	80.6678	79.7778	4.5983	0.9070
189	0.9868	138	0.1618	80.6678	80.6901	4.5983	0.9122
189	0.9868	137	0.1456	80.6678	81.6071	4.5983	0.9170
189	0.9868	136	0.1294	80.6678	82.5284	4.5983	0.9214
189	0.9868	135	0.1132	80.6678	83.4537	4.5983	0.9253
190	1.0029	134	0.0971	90.0000	84.4725	9.3322	1.0188
190	1.0029	133	0.0809	90.0000	85.3894	9.3322	0.9169
190	1.0029	132	0.0647	90.0000	86.3086	9.3322	0.9193
190	1.0029	131	0.0485	90.0000	87.2298	9.3322	0.9212
190	1.0029	130	0.0324	90.0000	88.1524	9.3322	0.9226
190	1.0029	129	0.0162	90.0000	89.0760	9.3322	0.9236
190	1.0029	128	0.0000	90.0000	90.0000	9.3322	0.9240

How to Reach Us:

Home Page:
www.freescale.com

Web Support:
http://www.freescale.com/support

USA/Europe or Locations Not Listed:
Freescale Semiconductor, Inc.
Technical Information Center, EL516
2100 East Elliot Road
Tempe, Arizona 85284
+1-800-521-6274 or +1-480-768-2130
www.freescale.com/support

Europe, Middle East, and Africa:
Freescale Halbleiter Deutschland GmbH
Technical Information Center
Schatzbogen 7
81829 Muenchen, Germany
+44 1296 380 456 (English)
+46 8 52200080 (English)
+49 89 92103 559 (German)
+33 1 69 35 48 48 (French)
www.freescale.com/support

Japan:
Freescale Semiconductor Japan Ltd.
Headquarters
ARCO Tower 15F
1-8-1, Shimo-Meguro, Meguro-ku,
Tokyo 153-0064
Japan
0120 191014 or +81 3 5437 9125
support.japan@freescale.com

Asia/Pacific:
Freescale Semiconductor Hong Kong Ltd.
Technical Information Center
2 Dai King Street
Tai Po Industrial Estate
Tai Po, N.T., Hong Kong
+800 2666 8080
support.asia@freescale.com

For Literature Requests Only:
Freescale Semiconductor Literature Distribution Center
P.O. Box 5405
Denver, Colorado 80217
1-800-441-2447 or 303-675-2140
Fax: 303-675-2150
LDCForFreescaleSemiconductor@hibbertgroup.com

freescale™
semiconductor

AN3461
Rev. 6
06/2007